Paper Reference(s) 6677/01 Edexcel GCE

Mechanics M1

Advanced Subsidiary

Monday 13 May 2013 – Afternoon

Time: 1 hour 30 minutes

<u>Materials required for examination</u> Mathematical Formulae (Pink) <u>Items included with question papers</u> Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M1), the paper reference (6677), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$. When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided. Full marks may be obtained for answers to ALL questions. There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled. You must show sufficient working to make your methods clear to the Examiner. Answers without working may not gain full credit.

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- 1. Particle *P* has mass 3 kg and particle *Q* has mass *m* kg. The particles are moving in opposite directions along a smooth horizontal plane when they collide directly. Immediately before the collision, the speed of *P* is 4 m s⁻¹ and the speed of *Q* is 3 m s⁻¹. In the collision the direction of motion of *P* is unchanged and the direction of motion of *Q* is reversed. Immediately after the collision, the speed of *P* is 1 m s⁻¹ and the speed of *Q* is 1.5 m s⁻¹.
 - (a) Find the magnitude of the impulse exerted on P in the collision.

(3)

- (b) Find the value of m. (3)
- 2. A woman travels in a lift. The mass of the woman is 50 kg and the mass of the lift is 950 kg. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration of 2 m s⁻². By modelling the cable as being light and inextensible, find
 - (*a*) the tension in the cable,

(3)

(b) the magnitude of the force exerted on the woman by the floor of the lift.

(3)





A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle α , where tan $\alpha = \frac{3}{4}$, and the plane is at an angle of 30° to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

(8)

4. A lorry is moving along a straight horizontal road with constant acceleration. The lorry passes a point A with speed $u \text{ m s}^{-1}$, (u < 34), and 10 seconds later passes a point B with speed 34 m s^{-1} . Given that AB = 240 m, find

<i>(a)</i>	the value of <i>u</i> ,	
		(3)

- (b) the time taken for the lorry to move from A to the mid-point of AB.
- 5. A car is travelling along a straight horizontal road. The car takes 120 s to travel between two sets of traffic lights which are 2145 m apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for 30 s until its speed is 22 m s^{-1} . The car maintains this speed for *T* seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.
 - (a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights.

(3)

(2)

(6)

A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, $a \text{ m s}^{-2}$, and passes the car at the point A which is 990 m from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed 22 m s⁻¹.

(*c*) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point *A*.

(*d*) Find the value of *a*.

(2)

(4)

- 6. A beam *AB* has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points *P* and *Q*, where AP = 2 m and QB = 3 m. When a child of mass 50 kg stands on the beam at *A*, the beam remains in equilibrium and is on the point of tilting about *P*. When the same child of mass 50 kg stands on the beam at *B*, the beam remains in equilibrium and is on the point of tilting about *Q*. The child is modelled as a particle and the beam is modelled as a non-uniform rod.
 - (a) (i) Find the mass of the beam.
 - (ii) Find the distance of the centre of mass of the beam from *A*.

(8)

(6)

When the child stands at the point X on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find AX.

7. [In this question, the horizontal unit vectors **i** and **j** are directed due east and due north respectively.]

The velocity, $\mathbf{v} \text{ m s}^{-1}$, of a particle *P* at time *t* seconds is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (3t - 3)\mathbf{j}.$$

(a) Find the speed of P when t = 0.

(b) Find the bearing on which P is moving when t = 2.

- (c) Find the value of t when P is moving
 - (i) parallel to **j**,
 - (ii) parallel to $(-\mathbf{i} 3\mathbf{j})$.

(6)

(3)

(2)





Two particles A and B have masses 2m and 3m respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a smooth horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the table. Particle B hangs at rest vertically below the pulley with the string taut, as shown in Figure 2. Particle A is released from rest. Assuming that A has not reached the pulley, find

(<i>a</i>)	the acceleration of <i>B</i> ,	(5)
(<i>b</i>)	the tension in the string,	(-)
(c)	the magnitude and direction of the force exerted on the pulley by the string	(1)
	the magnitude and anecdon of the force excited on the puncy by the sump.	(4)

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.		
(a)	For <i>P</i> , $-I = 3(1-4)$	M1 A1
	I = 9 Ns	A1
		(3)
(b)	For Q , $9 = m(1.53)$	M1 A1
	m = 2	A1
	OR	
	12 - 3m = 3 + 1.5m	M1 A1
	m=2	A1
		(3)
		[6]
	Notes for Question 1	
Q1(a)	be considering <i>one</i> particle i.e. have <i>same mass</i> in both terms) (M0 if g is included or if mass omitted). First A1 for $\pm 3(1-4)$ Second A1 for 9 (Must be positive). Allow change of sign at end to	
	obtain magnitude. <u>N.B.</u> For M1 they may use CLM to find a value for m first and then use it when considering the change in momentum of Q to find the impulse.	
Q1(b)	EITHER M1 for attempt at: their Impulse from (a) = difference in momenta for particle Q , (must be considering <i>one</i> particle) (M0 if g is included or if mass omitted). First A1 for $9 = m(1.53)$ oe. Q1(b) Second A1 for $m = 2$.	
	OR M1 for attempt at CLM equation, with correct no. of terms, dimensionally correct. Allow consistent extra g's and sign errors. First A1 for a correct equation i.e. $12 - 3m = 3 + 1.5m$ oe. Second A1 for $m = 2$.	

Question Number	Scheme	Marks
2.		
(a)	For system, (1), $T - 950g - 50g = 1000 \times -2$	M1 A1
	T = 7800 N	Al
		(3)
(b)	For woman, (\uparrow) , $R-50g = 50 \times -2$	M1 A1
	R = 390 N	Al
		(3)
		[6]
	Notes for Question 2	
(In both parts, use the <i>mass</i> to decide which part of the system is being		
	considered and M marks can only be scored if an equation contains only	
	forces acting on that part of the system)	
Q2(a)	M1 is for a complete method for finding <i>T</i> i.e. for an equation in <i>T</i> only,	
	dimensionally correct, with the correct number of terms.	
	First A1 for a correct equation.	
	Second A1 for 7800 (N).	
	M1 is for a complete method for finding <i>R</i> i.e. for an equation in <i>R</i> only,	
	dimensionally correct, with the correct number of terms.	
Q2(b)	First A1 for a correct equation.	
	Second A1 for 390 (N).	
	N.B. Equation for lift <i>only</i> is: $T - 950g - R = 950 \text{ x} (-2)$	

Question Number	Scheme	Marks
3.	$T\cos\alpha - F = 2g\cos 60^{\circ}$	M1 A1
	$T\sin\alpha + R = 2g\cos 30^{\circ}$	M1 A1
	$F = \frac{1}{3}R$	B1
	eliminating F and R	DM 1
	$T = g(1 + \frac{1}{\sqrt{3}}), 1.6g \text{ (or better)}, 15.5, 15 \text{ (N)}$	DM 1 A1
		(8)
		[8]
	Notes for Question 3	
	First M1 for resolving norallal to the plane with correct no. of terms and	
Q3	First M1 for resolving parallel to the plane with correct ho. of terms and both <i>T</i> and 2 <i>g</i> terms resolved. First A1 for a correct equation. (use of α instead of 30° or 60° or vice versa is an A error not M error; similarly if they use $\sin(3/5)$ or $\cos(4/5)$ when resolving, this can score M1A0) Second M1 for resolving perpendicular to the plane with correct no. of terms and both <i>T</i> and 2 <i>g</i> terms resolved. Second A1 for a correct equation (use of α instead of 30° or 60° or vice versa is an A error not M error; similarly if they use $\sin(3/5)$ or $\cos(4/5)$ when resolving, this can score M1A0) B1 for $F = 1/3 R$ seen or implied. Third M1, dependent on first two M marks and appropriate angles used when resolving in <i>both</i> equations, for eliminating <i>F and R</i> . Fourth M1 dependent on third M1, for solving for <i>T</i> Third A1 for 15(N) or 15.5 (N). N.B. The first two M marks can be for two resolutions in any directions. Use of tan $\alpha = 4/3$ leads to an answer of 17.83 and can score may 7/8	

Question Number	Scheme	Marks
4.		
(a)	$240 = \frac{1}{2}(u+34)10$	M1 A1
	u = 14	A1
		(3)
(b)	$34 = 14 + 10a \implies a = 2$	M1 A1
	$120 = 14t + \frac{1}{2} \times 2 \times t^2$	M1 A1
	$t^2 + 14t - 120 = 0$	
	Solving, $t = -20$ or 6	DM 1
	t = 6	A1
	OR	
	$34 = 14 + 10a \implies a = 2$	M1 A1
	$v^2 = 14^2 + 2 \times 2 \times 120 \implies v = 26$	
	AND $26 = 14 + 2t$	M1 A1
	t = 6	DM 1 A1
		(6)
		[9]
	Notes for Question 4	
	First M1 for a complete method to produce an equation in u only.	
Q4(a)	First A1 for a correct equation. $(u^2 - 48u + 4/6 = 0$ oe is possible). Second A1 for $u = 14$.	
04 (b)	EITHER First M1 for an equation in <i>a</i> only. (M0 if $v = 34$ when $s = 120$ is used) First A1 for $a = 2$. (This may have been found in part (a)) Second M1 for a 3-term quadratic equation in <i>t</i> only, allow sign errors (must have found a value of <i>a</i> . (M0 if $v = 34$ when $s = 120$ is used) Second A1 for a correct equaton. Third M1 dependent on previous M1 for solving for <i>t</i> . Third A1 for $t = 6$	
עיש	OR First M1 for an equation in <i>a</i> only. First A1 for $a = 2$. (This may have been found in part (a)) Second M1 for a complete method to obtain an equation in <i>t</i> only, allow sign errors. (must have found a value of <i>a</i>) Second A1 for a correct equaton. Third M1 dependent on previous M1 for solving for <i>t</i> . Third A1 for $t = 6$	

Question	Scheme	Marks
Number		
5.	Spood	D1
(a)	Speed Shape	DI D1
	22 Figures	
		(2)
	0 30 30+7 120 Time	
(b)	$\frac{(120+T)22}{2} = 2145$	M1 A1
	T = 75	A1
		(3)
(c)	$\frac{(t+t-30)22}{2} = 990$	M1 A1
	t = 60	A1
	Answer = 60 - 10 = 50	A1
		(4)
(d)	$990 = 0.5a50^2$	M1
	a = 0.79, 0.792, 99/125 oe	A1
		(2)
		[11]
Notes for Question 5		
Q5(a)	First B1 for a trapezium starting at the origin and ending on the <i>t</i> -axis. Second B1 for the figures marked (allow missing 0 and a delineator of for <i>T</i>) (allow if they have used $T = 75$ correctly on their graph)	
	First M1 for producing an equation in their $T only$ by equating the area	
Q5(b)	First M1 for producing an equation in their <i>T</i> only by equating the area of the trapezium to 2145, with the correct no. of terms. If using a single trapezium, we need to see evidence of using $\frac{1}{2}$ the sum of the two parallel sides or if using triangle(s), need to see $\frac{1}{2}$ base x height. Second A1 cao for a correct equation in <i>T</i> (<u>This is not f.t. on their <i>T</i></u>) Third A1 for <i>T</i> = 75. N.B. Use of a single <i>suvat</i> equation for the whole motion of the car e.g. $s = t(u+v)/2$ is M0	
Q5(c)	First M1 for producing an equation in <i>t</i> only (they may use $(t - 30)$ oe as their variable) by equating the area of the trapezium to 990, with the correct no. of terms. If using a trapezium, we need to see evidence of using $\frac{1}{2}$ the sum of the two parallel sides or if using triangle(s), need to see $\frac{1}{2}$ base x height. First A1 for a correct equation. Second A1 for $t = 60$ (Allow $30 + 30$). Third A1 for answer of 50. N.B. Use of a single <i>suvat</i> equation for the whole motion of the car e.g. $s = t(u+v)/2$ is M0. Use of the motion of the motorcycle is M0 (insufficient information). Use of $v = 22$ for the motorcycle is M0.	
Q5(d)	First M1 for an equation in <i>a</i> only. First A1 for $a = 0.79$, 0.792, 99/125 oe N.B. Use of $v = 22$ for the motorcycle is M0.	

Question Number	Scheme		Marks	
6.				
(a)	P		QP	
	⁷⁴ 2 m ▲		▲ 3 m ^D	
	, 			
		▼ Mg		
	<	•		
	A 111			
	$M(P), \qquad 50g \times 2$	$=Mg \times (x-2)$		M1 A1
	$M(Q)$, $50g \times 3$	$= Mg \times (12 - x)$		M1 A1
(i)	M = 25 (kg)			DM 1 A1
(ii)	x = 6 (m)			DM 1 A1
				(8)
(b)	P		0	
	A 2 m 🛉	X	▲3 m B	
	l I R		l R	
		50a		
		50g		
	$(\uparrow)R + R = 25g + 5$	0g		M1 A1 ft
	M(A), $2R + 12R =$	$25g \times 6 + 50g \times AX$		M1 A1 ft
	AX = 7.5 (m)			DM 1 A1
				(6)
				[14]

Notes for Question 6		
Q6(a)	First M1 for moments about P equation with usual rules (or moments about a different point AND vertical resolution and R then eliminated) (M0 if non-zero reaction at Q) Second M1 for moments about Q equation with usual rules (or moments about a different point AND vertical resolution) (M0 if non-zero reaction at P) Second A1 for a correct equation in M and same unknown. Third M1, dependent on first and second M marks, for solving for M Third A1 for 25 (kg) Fourth M1, dependent on first and second M marks, for solving for x Fourth A1 for 6 (m) <u>N.B. No marks available if rod is assumed to be uniform but can score</u> max 5/6 in part (b), provided they have found values for M and x to f.t. <u>on</u> . If they have just invented values for M and x in part (a), they can score the M marks in part (b) but not the A marks	
Q6(b)	First M1 for vertical resolution or a moments equation, with usual rules. First A1 ft on their <i>M</i> and <i>x</i> from part (a), for a correct equation. (must have <i>equal reactions</i> in vertical resolution to earn this mark) Second M1 for a moments equation with usual rules. Second A1 ft on their <i>M</i> and <i>x</i> from part (a), for a correct equation in <i>R</i> and same unknown length. Third M1, dependent on first and second M marks, for solving for <i>AX</i> (not their unknown length) with $AX \le 15$ Third A1 for $AX = 7.5$ (m) N.B. If a single equation is used (see below), equating the sum of the moments of the child and the weight about <i>P</i> to the sum of the moments of the child and the weight about <i>Q</i> , this can score M2 A2 ft on their <i>M</i> and <i>x</i> from part (a), provided the equation is in one unknown. Any method error, loses both M marks. e.g. $25g.4 + 50g(x - 2) = 25g.6 + 50g(12 - x)$ oe.	

Question Number	Scheme	Marks
7.		
(a)	$t = 0$ gives $\mathbf{v} = \mathbf{i} - 3\mathbf{j}$	B1
	speed = $\sqrt{1^2 + (-3)^2}$	M1
	$=\sqrt{10} = 3.2$ or better	A1
		(3)
(b)	$t=2$ gives $\mathbf{v} = (-3\mathbf{i}+3\mathbf{j})$	M1
	Bearing is 315°	A1
		(2)
(c)(i)	$1 - 2t = 0 \Longrightarrow t = 0.5$	M1 A1
(ii)	-(3t-3) = -3(1-2t)	M1 A1
	Solving for <i>t</i>	DM 1
	t = 2/3, 0.67 or better	A1
		(6)
		[11]
	Notes for Question 7	1
	BI for $\mathbf{i} - 3\mathbf{j}$.	
Q7(a)	M1 for $\sqrt{(\text{sum of squares of cpt.s)}}$	
	A1 for $\sqrt{10}$, 5.2 of better M1 for alage attempt to sub $t = 2$ into given expression	
Q7(b)	A 1 for 315	
	(i) First M1 for $1 - 2t = 0$	
	First A1 for $t = 0.5$	
	N.B. If they offer two solutions, by equating both the i and j	
	components to zero, give M0.	
	1 - 2t - 1	
Q7(c)	(ii) First M1 for $\frac{3t-3}{3t-3} = \pm (\frac{-3}{-3})$ o.e. (Must be an equation in t	
	only)	
	First $A1$ for a correct equation (the + sign)	
	Second M1, dependent on first M1, for solving for t.	
	Second A1 for 2/3, 0.67 or better.	

Question Number	Scheme	Marks
8.		
(a)	For A , $T = 2ma$	B1
	For B , $3mg - T = 3ma$	M1 A1
	3mg = 5ma	DM 1
	$\frac{3g}{5} = a$ (5.9 or 5.88 m s ⁻²)	A1
		(5)
(b)	T = 6mg/5; 12m; 11.8m	B1
		(1)
(c)	$F = \sqrt{T^2 + T^2}$	M1 A1 ft
	$F = \frac{6mg\sqrt{2}}{5}; 1.7mg \text{ (or better)}; 16.6m; 17m$	A1
	Direction clearly marked on a diagram, with an arrow, and 45° (oe) marked	B1
		(4)
		[10]
	Notes for Question 8	
	B1 for $T = 2ma$	
	First M1 for resolving vertically (up or down) for B , with correct no. of terms. (allow omission of m , provided 3 is there) First A1 for a correct equation.	
Q8(a)	Second M1, dependent on first M1, for eliminating <i>T</i> , to give an equation in <i>a</i> only. Second A1 for 0.6g, 5.88 or 5.9. N.B. 'Whole system' equation: $3mg = 5ma$ earns first 4 marks but any error loses all 4.	
Q8(b)	B1 for $\frac{6mg}{5}$, 11.8 <i>m</i> , 12 <i>m</i>	
Q8(c)	M1 $\sqrt{(T^2 + T^2)}$ or $\frac{T}{\sin 45^\circ}$ or $\frac{T}{\cos 45^\circ}$ or $2T\cos 45^\circ$ or $2T\sin 45^\circ$ (allow if <i>m</i> omitted) (M0 for $T\sin 45^\circ$) First A1 ft on their <i>T</i> .	
	Second A1 cao for $\frac{-6.1}{5}$ oe, $1.7mg$ (or better), $16.6m$, $17m$ B1 for the direction clearly shown on a diagram with an arrow and 45° marked.	